

# How Financial Transmission Rights Curb Market Power

*Steven Stoft*

Environmental Energy Technologies Division  
Ernest Orlando Lawrence Berkeley National Laboratory  
University of California  
Berkeley, California 94720

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## **Abstract**

This paper demonstrates that financial transmission rights allow their owners to capture at least a portion, and sometimes all, of the congestion rents. This extends work in this area by Shmuel Oren which was limited to the case in which generators could not purchase financial transmission rights. One form of financial rights, Transmission Congestion Contracts (TCCs), is shown to be so effective in reducing market power that as few as two generators facing a demand curve with zero elasticity may be forced to sell at marginal cost. The extent to which market power is limited depends on the extent to which total generation capacity exceeds export capacity and on the size of individual generators. A relationship is derived that determines when TCCs will eliminate market power. In the case of a three line network, it is shown that the reduction in market power that can be accomplished with “active transmission rights” can also be accomplished with simple contracts for differences.



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## Executive Summary

This paper reconstructs the analysis of Transmission Congestion Contracts (TCCs) under the zero-elasticity (ZE) conditions analyzed by Shmuel Oren in his January 1997 *Energy Journal* article. The ZE condition is rare but does occur when generation but not load is located at a network node serviced by at least one congested line. Oren concluded that in a zero-elasticity setting without “active” transmission rights: (1) the market value of TCCs is zero; and (2) generators capture all congestion rents no matter how competitive their market. These conclusions, if correct, would indicate dramatic and problematic market power under such conditions. He also concluded that this problem could only be alleviated by “active,” i.e., “physical” transmission rights.

Fortunately, none of these conclusions is correct. The primary flaw in his analysis was his assessment of the “market value of TCCs” without including a market for TCC’s. When this market is included, TCCs take on value as hedging instruments for generators just as they were intended to and as Oren has recognized they would in previous work (1994). Oren does introduce a market for “active” rights and finds them valuable. Analogously, by introducing a market for TCCs we find that:

- ! TCCs have positive market value.
- ! Even with only two competing generators, if there is enough excess capacity, they will lose all market power and TCCs will take on full value.
- ! With any fixed level of excess capacity, generators lose all market power and TCCs take on full value as the generation market becomes competitive.
- ! In Oren’s three-node network example, arbitrage will limit market power and give TCCs value even without any market for TCCs or active rights.

The only case in which Oren’s results are sustained is the one-line zero-elasticity model without a market for any type of transmission rights. In this case, Oren has correctly shown that none of the Nash equilibria provide a competitive outcome. However, even in this case, both logic and experimental evidence indicate that a large number of generators will find it impossible to settle on any particular equilibrium out of Oren’s infinite set.

Two broad conclusions can be drawn. First, the problem that Oren has pointed out is extremely limited. It applies only to zero-elasticity nodes, and there is as yet no evidence that it occurs outside of Oren’s one-line model. Second, TCCs are very effective at curbing generation market power in this one case where they are needed. There is no indication that they would ever be less effective than active rights. Lastly it should always be remembered that no market structure is likely to solve all market power problems.





# 1 Introduction

The problem of allocating scarce transmission resources continues to be a central issue in the process of restructuring the electricity industry. The Federal Energy Regulatory Commission has issued an interim ruling (Order 888) and simultaneously a notice of a proposed new rule (the so-called CRT NOPR). While there is no end to new proposed solutions, the central dichotomy remains: should the rights be physical or financial? The most recent contribution to this debate is an article by Shmuel Oren in *The Energy Journal* (1997a). In this piece, he raises the interesting and important question of how transmission rights interact with the market power of generators which is sometimes created by congested lines. The present paper extends that analysis to a more realistic representation of financial transmission rights that allows them to be purchased by the generators in question. The result of this extension is to demonstrate the previously unnoticed ability of financial transmission rights to curb market power and to partially characterize this ability.

Oren has labeled financial rights, and transmission congestion contracts (TCCs) in particular, as “passive” and has labeled what are usually called physical rights as “active.” Most often he characterizes “passive” as meaning rights “that are compensated ex-post based on nodal prices,” (p. 63) but sometimes he adds the additional characterization that there is no “trading of rights” (p. 65). We will find that the first characterization is irrelevant but that the second matters a great deal. He characterizes active rights as tradable and, because of obvious market-power problems, rejects the notion that owners of active rights could de facto withdraw them from the market.<sup>1</sup> All of these characterizations are accurate except for the description of TCCs as being non-tradeable. While the non-tradeability assumption may be convenient for preliminary analysis, the trading of TCCs was intended from their conception, was recognized by Oren as early as 1994 and as late as June 1997, and is unpreventable in practice.<sup>2</sup> The present analysis removes this unrealistic assumption and reworks Oren’s analysis of generator market power and congestion rents accordingly.

Oren concludes that because “congestion rents will be captured by generators,” (p. 65) “Passive owners of transmission rights in the form of TCCs ... will be disappointed to discover that no rents have been accrued.” We will find that because TCCs are tradeable this conclusion is generally not warranted, and that the exact opposite can be true. This is fully consistent with the results of Harvey, Hogan, and Pope (1996). Instead of generators capturing congestion rents from transmission owners, transmission owners will curb the market power of generators as they capture rents for themselves. We explore the limits on their ability to curb this market power and find that it depends on the excess of total

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<sup>1</sup> This position was clearly articulated in Oren (1997b) p. 31.

<sup>2</sup> In his *Electricity Journal* article (1997b) Oren says, “while TCCs can be traded. . . .”

generation capacity over line capacity and on the size of generators. In fact, given sufficient excess capacity, even two generators will bid prices down to marginal cost at a congested node. This result is quite reassuring given Oren's prediction that, without TCC trading, no matter how many generators compete and no matter how great the excess of generation capacity, generators will raise their bids above marginal costs to capture the full congestion rent.

### *1.1 Zero-Elasticity Demand*

Oren has usefully brought to our attention the problem of zero-elasticity demand in a congested network. Given a transmission line from a generation node to a demand node, if the line is being used to its capacity limit, then a decrease in price cannot call forth any more demand at the generation node. Similarly, as long as the generation price stays lower than the price at the demand node, an increase in price will not decrease effective demand because desired demand will continue to exceed transmission capacity.

It is important to note that the phenomenon of zero elasticity only occurs when there is no demand at the generation node and all lines out of that node are congested. Consequently its occurrence may be quite rare. Nonetheless it could occur, and it makes an excellent test case for the study of market power.

Without retracing Oren's mathematics, we can grasp his result intuitively by referring to a well-know fact about Cournot competition. That is, markup is inversely proportional to demand elasticity, so for zero elasticity we have infinite markup. Of course the outcome will not be an infinite markup because as soon as generation-node price gets as high as the demand-node price, demand will become elastic. If it becomes elastic enough, then that will prevent any further price increase.

As we will see this result leaves open the question of how much each generator produces and thus how much each profits. The resulting plethora of Nash equilibria leaves the actual solution of the game in doubt, but for the most part we will assume for the sake of exposition that Oren is right in predicting that some generators capture the congestion rent.

### *1.2 The Introduction of TCCs*

Transmission congestion contracts are one specific form of financial transmission right. A TCC gives its owner the right to collect the congestion rent between two nodes for a specific amount of time and for some contract-specified power flow. For instance a TCC might specify 1 MW of flow from Bus 1 to Bus 2 during certain peak hours for the next year. For

simplicity, we will assume that TCCs have balanced inputs and outputs, and we will ignore losses.

To understand congestion rents, we must introduce nodal (or equivalently zonal) pricing. Nodal pricing allows generators and loads to bid supply or demand curves at their own bus and then clears the market subject to all transmission constraints. If the independent system operator (ISO) imposes only the required constraints and if the traders bid their marginal cost and valuation curves, an efficient set of prices is produced. Ignoring the infamous “loop flows,” the price will be low at the inflow end of a congested line and high at the outflow end. This price difference is the congestion rent. Since the ISO essentially buys power at the inflow end and re-sells it at the outflow end, the ISO collects the congestion rent.

To understand TCCs, consider someone who owns a TCC from Bus 1 to Bus 2 that is equal to half the line capacity. If the price at Bus 1 is low and at Bus 2 is high because of congestion on the line, then the TCC owner will collect from the ISO half of the congestion rent that the ISO collects on that line. The purpose of TCCs was well described by Oren (1994) as follows:

“Second, it provides users of the grid a mechanism to hedge against congestion by purchasing transmission rights.”<sup>3</sup>

This paper will investigate the ramifications of just such hedging. We will ask and answer the question: What would happen if TCCs were introduced into Oren’s zero-elasticity market model and the generators purchased them as a mechanism for hedging against congestion, as he has suggested they should.

### 1.3 Organization

The next section reviews the analysis of the zero-elasticity, energy-only model and then reinterprets its infinite set of Nash equilibria. Rivalry over these equilibria give value to anything that confers a bargaining advantage. Section 3 shows that TCCs are ideally suited to the role and can easily absorb the entire value of the congestion rent if there is a tradeable supply of them equal to the transmission capacity. Section 4 discusses two reasons that TCC owners may fail to capture all of the congestion rent: the first is limited generation capacity; and the second is limited TCC availability. Section 5 shows that a three-line network is less problematic than a one-line network. Only contracts for differences are needed to arbitrage away generator market power and restore the value of TCCs. Section 6 concludes and the

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<sup>3</sup> Oren, Shmuel, P. Spiller, P. Variaya, F. Wu. “Nodal Prices and Transmission Rights: A Critical Appraisal.” POWER Working Paper, PWP-025, December, 1994.

appendix discusses two possible objections to the present analysis. The first of these is the embedded in Oren's use of non-standard game theory. The second is the possibility of a faulty TCC market.

## 2 The Zero-Elasticity Model

We will consider a simple one-line, zero-elasticity model. It has generators at Bus 1 and load at Bus 2. An important feature of this model is that demand is entirely absent at Bus 1. For simplicity we will assume that there is competitive generation as well as demand at Bus 2, and that these generators are operating in a flat section of their marginal cost curve. As a consequence, any reduction in supply from Bus 1 will be met by an equal increase in output at Bus 2, and the price at Bus 2 will stay constant at the marginal cost of generation which we will take to be \$20/MW.

Bus 2 can now be characterized simply by its price and the fact that it will absorb any output from Bus 1 that can be made available over the line. We will assume this line has a capacity limit of  $K = 4$  MW and that there are only two generators, each with a marginal cost of \$15/MWh and no fixed costs, as is shown in **Figure 1**.<sup>4</sup>

To analyze the competition between the two generators we choose the Cournot model recast as is now standard as a strategic game, and we will seek the Nash equilibria of this game. In this form, the strategies of the competing firms consist simply of their outputs which are chosen simultaneously. The price is determined by the intersection of the aggregate supply function and the aggregate demand function. The supply function is assumed to be zero out to the sum of the specified quantities and infinite thereafter. This is shown in **Figure 2**. The demand function is zero for prices above the

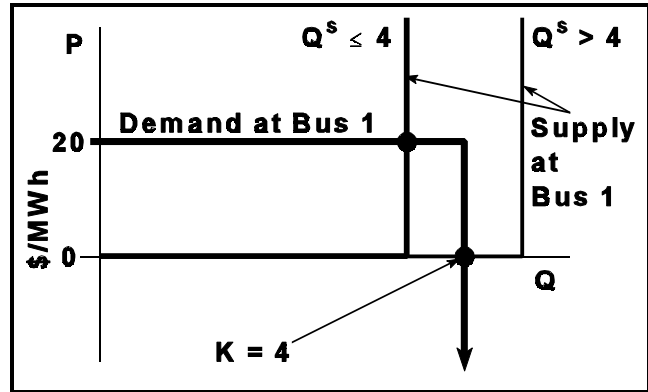


Figure 1. Cournot Supply Functions and Zero-Elasticity Demand Over a Constrained Line.

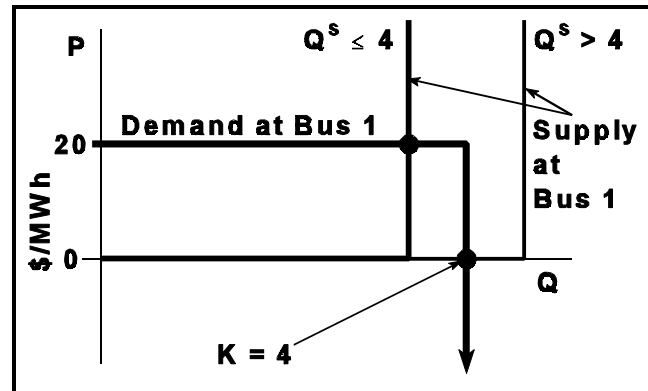


Figure 2. Cournot Supply Functions and Zero-Elasticity Demand Over a Constrained Line.

<sup>4</sup> This paper will generally adopt the approach of working with specific examples and assuming the interested reader can generalize without difficulty to other line sizes, marginal costs and numbers of firms. This minimizes notational complexity.

demand-node price and equal to the line capacity for lower prices. This also is shown in **Figure 2** for the specific values of our example.

As can be seen in **Figure 2**, when the quantity supplied,  $Q^S$ , is less than the line capacity,  $K$ , the price paid to Bus-1 generators will be \$20/MWh, the price at Bus 2. This is consistent with pricing on an unconstrained network. If the total quantity supplied by generators at Bus 1 is greater than 4, then the price at Bus 1 is zero. We resolve the pricing ambiguity when  $Q^S=K$  by defining price to be \$20/MWh in this case.<sup>5</sup> This can be expressed algebraically as follows:

**The Bus-1, Energy-Price Determination Rule:**

$$\begin{aligned} p &\ni 20 && \text{if } q_1 \leq q_2 \leq 4 \\ p &\ni 0 && \text{if } q_1 \leq q_2 > 4 \end{aligned} \tag{1}$$

Another approach would be to allow supply-curve bidding on the part of the generators. They could then use bids of the form: \$p/MWh for a supply of  $q \leq k$  and no supply greater than  $k$  at any price. These more realistic bids would, in most cases, result in the same outcome and the same analysis.

We have now specified price when the two generators supply  $q_1+q_2 > 4$ , but we have not specified how the line constraint is met in this circumstance. A plausible model might specify that the ISO restricts the outputs of the two generators in proportion to their bids. We will simply assume that the ISO simply disposes of excess power. This makes little difference since at a price of zero generators will try hard to avoid this outcome in any case.

Lastly we will specify the output capacity of the two generators as  $\bar{q}_1$  and  $\bar{q}_2$ , which we will assume in our initial example to be 4MW each. Once TCCs are introduced, this assumption, will prove to be the crucial assumption governing the exercise of market power by the two generators.

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<sup>5</sup> Economics does not resolve the price ambiguity when the supply is exactly 4, but a resolution will be necessary both for practical operation and to construct a well-defined model. In practice how this infinitesimal distinction is treated makes no difference because a firm can cause supply to cross this boundary at essentially zero cost. From a modeling perspective it is important that a best-response strategy set always exist. Because defining  $p$  to be less than 20 if  $Q^S=4$  implies there is no largest  $Q^S$  for which  $p=20$ , we define  $p$  to equal 20 in this case.

## 2.1 The Nash Equilibrium

The Cournot model is a non-cooperative game so, according to Nash's theorem, it must have at least one Nash equilibrium.<sup>6</sup> If it has only one such equilibrium, which it typically does, this equilibrium can reasonably be considered the "solution" to the game in the sense that rational players would adopt the strategies specified in the Nash equilibrium. Consequently we begin our analysis of the Cournot game by looking for Nash equilibria. To make it easy to check potential equilibria we specify algebraically this game which we will call the energy-only game.

### The Energy-Only Game

Strategies:  $0 \leq q_i \leq 4$  where  $i = 1, 2$

Payoff functions:  $B_i(q_i | q_j) = (p - 15)q_i$  for  $i = j$  (2)

where:  $p$  is determined by the Price Determination Rule, equation (1)

As is well known, in a Nash equilibrium each player finds its strategy optimal given that the other's strategy is fixed. This means that each of the two strategies in a Nash equilibrium must solve the optimization problem:

$$\max_{q_i} B(q_i | q_j) \text{ for } i = j \quad (3)$$

Having specified the game and the Nash condition, it is easy to see that there are an infinity of Nash equilibria that can be characterized as follows. The strategy pair  $(q_i, q_j)$  is a Nash equilibrium if and only if  $q_i, q_j \leq 4$  and  $q_i + q_j = 4$ . In other words, any combination of possible outputs that fills the line but does not congest it is a Nash equilibrium. (I will say a line is congested if and only if curtailment or dumping of power is required.) These are all Nash equilibria because if either generator produces less it fails to earn \$5/MWh profit on the reduced production, and if it produces more it causes the price to drop to zero. This is the same set of equilibria found by Oren, and we have no disagreement with this finding.

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<sup>6</sup> This is true so long the strategy space is finite, which is the case if  $q$  is restricted to, say, an integer number of kW. In fact we will shortly find a NE for the game with a continuum of strategies. This is guaranteed because the strategy set is compact and the response function is continuous.



## 2.2 *Interpreting the Nash Equilibrium*

When a game has two equilibria, it is no longer clear that either one of the two is a good description of how the game will be played. In the case of an infinite number of Nash equilibria the problem is compounded. If we consider any one particular Nash equilibrium (NE), say  $q_1 = 1$ ,  $q_2 = 3$ , we find that both players have other equilibria that they much prefer. If Generator 1 could force a change to the equilibrium that includes  $q_1 = 3$ , it would triple its profit. Clearly anything that would give any player an advantage in determining which NE is realized will be highly valued by the players.

Oren assumes that on every play of such a game one pairs of NE strategies will be played, but he makes no argument for this assumption. What is to prevent Generator 1 from producing an output of 3 in order to attempt to impose the (3,1) NE, while Generator 2 insists on an output of 3 in order to impose the (1,3) NE? Unfortunately, as is well known, game theory provides no definitive answer to this question. But let us, for the sake of argument, assume that Oren has guessed right and that the market will always produce one of the NEs. In this case he is also right that the price at Bus 1 will be \$20/MWh and that any TCC from Bus 1 to Bus 2 will be worthless. This is Oren's result in a nutshell.

Clearly this result generalizes to any number of supply generators. Thus, even with 100 generators at Bus 1, competition will fail to bring down the price even a little. The only thing that may change as the number of generators increases is our comfort level with the assumption that no generator, dissatisfied with its position in the reigning NE, would ever break ranks and stubbornly increase output in an attempt to shift the game to a more favorable NE. This extreme prediction has already been contradicted by early experiments which show generators capturing only a part of the congestion rent, and could easily be tested definitively by using many players at the end of the line and giving them a large amount of excess capacity.<sup>7</sup> I predict that such an experiment would dramatically disconfirm Oren's prediction.

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<sup>7</sup> See Backerman, Rassenti and Smith, 1996.

### 3 Enter TCCs

We now move into fresh territory by introducing a market for TCCs. In Section 1 we concluded that, in the zero elasticity model, generators would capture all congestion rent and TCCs would be worthless. The crucial assumption was that TCCs could not be purchased by generators. In short, there was no market for TCCs. Because of this restriction, Oren's claim that this model establishes the "market value of TCCs" seems premature.<sup>8</sup> Before drawing this conclusion it would seem necessary to introduce a market for TCCs and appropriate to ask what a generator would be willing to pay for a TCC. We now construct a model within which this question can be asked and answered.

As noted above, a primary purpose of TCCs is to allow users of the grid to hedge by purchasing transmission rights. In fact all TCC literature considers them tradable, and some goes so far as to consider the possibility that TCCs be traded to the point where their ownership perfectly matches use of the grid (Bushnell and Stoft, 1996). In fact they can be effectively traded without the consent of the ISO. If a TCC owner decides that she will not need this TCC between 1:00 and 2:00PM this afternoon and calls up a generator who might need a hedge at that time, they can assign on the spot all the TCC revenue earned during that hour to the generator. No permission is required for such a sale, and the contract will be legally binding. Thus, within our legal framework, it is literally impossible for TCCs not to be effectively tradable. Thus if TCCs have value to generators but not to the current TCC owners, one must expect that a market in TCCs would soon develop.<sup>9</sup>

To find the value of TCCs to generators we need to model a market in which they can be purchased by generators. There are many forms such a market could take, and the price of TCCs could be reduced by a badly functioning market. But instead of attempting to analyze all possible TCC markets or to predict the actual market form, let us ask the simple question. What would TCCs be worth if the market functioned well? A basic tenant of free-market economics is that relatively efficient markets will develop when there is money to be made by trading in them. We assume nothing more. Although we will specify a very simple form for the TCC market, and one that is not likely to be realistic, any efficient market would provide the owners of TCCs with as good a price for their TCCs.<sup>10</sup>

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<sup>8</sup> (Oren, 1997a, p. 65)

<sup>9</sup> There could be more complex network configurations in which, to alleviate market power, it would be necessary to re-configure TCCs, if they were not suitable configured. This would require the help of the ISO. But no such examples have been discovered, and there is no particular reason to believe they exist.

<sup>10</sup> In fact the market we will analyze is probably quite efficient and may even assign TCCs their full value.

### 3.1 The TCC Market

Because auctions are relatively simple, we will imagine an auction in which the ISO holds all TCCs and auctions them periodically. The resulting revenue could be used to pay down transmission access charges. But these details are unimportant. All we need to specify are the rules under which the auction is conducted. For the bidding rule, let us specify that each generator will submit a price and quantity, say \$4/MWh for up to 3 MW of TCCs. We will represent this bid by the pair  $(P_i, t_i)$ . The auction could cover any time period, an hour or a year.

The bid evaluation rule will be to sell the TCCs to the highest bidder at the price offered. If there is a tie then the available TCCs will be split between the tying bidders in proportion to their bid quantities. Algebraically this is expressed as follows, where  $T_i$  is the quantity of TCCs sold to the  $i^{\text{th}}$  bidder. Note that  $i \neq j$ .

**The TCC Bid Evaluation Rule:**

$$\begin{aligned} T_i &\ni t_i && \text{if } P_i > P_j \\ T_i &\ni \min(t_i, 4 - t_j) && \text{if } P_i < P_j \\ T_i &\ni \min(t_i, 4 - t_j / (t_i + t_j)) && \text{if } P_i = P_j \end{aligned} \quad (4)$$

This specification is again for a two-generator game but could easily be generalized.

The timing of our new game, which we will call the “TCC game,” must also be specified. While we could specify that the two markets occur simultaneously, this is not realistic, and, as it turns out, it is not a simplification. For these reasons we specify that the TCC auction happens first and the energy auction second. This means that the bid,  $q$ , in the energy auction can depend on the outcome,  $T$ , of the TCC auction. This completes the specification of the TCC game, and we now present that specification algebraically as follows:

**The TCC Game**

$$\begin{aligned} \text{Strategies: } & (P_i, t_i, q_i(T_i)) \quad \text{where } 0 \leq q_i \leq 4, \quad 0 \leq t_i \leq 4, \quad \text{and } i \in \{1, 2\} \\ \text{Payoff functions: } & B_i(q_i | q_j) \ni (p + 15) - q_i & P_i > T_i \text{ or } (20 - p) - T_i \quad \text{for } i \neq j \end{aligned} \quad (5)$$

where:  $p$  is determined by Equation (1).

and where:  $T_i$  is determined by Equation (4).

Recall that  $T_i$  is the actual quantity of TCCs purchased in the auction while  $t_i$  is the bid quantity. The payoff (profit) function now includes the cost of the purchased TCCs and the revenue from these TCCs after the energy market settles,  $(20 - p)$  being the price difference between Bus 2 and Bus 1. We will often need to evaluate the payoff of strategy pairs when

checking for NEs, and we will do this by substitution into equation (5) with the order of all variables maintained for easy identification.

This specification of the game reduces it to strategic (normal) form from the extensive form that describes it as a sequence of two markets. In this form the players simultaneously choose a strategy, and the payoffs are then determined using the payoff functions. We specify the game in this form in order to apply the Nash equilibrium condition.

### 3.2 *The Unique Nash Equilibrium*

As with our previous game, the Nash equilibria are easy to spot. This time there is only one. Each player will choose  $(P=5, t=4, q=T)$ . Notice that  $(P=5, t=4, q=2)$  is not a NE even though the value of  $q$  in the true NE will be 2. The difference between these two strategies is best explained using the extensive form of the game in which the TCC market happens first. Choosing a strategy with  $q=T$  means choosing to produce only as much as is covered by one's successful purchase of TCCs. This strategy is harder for an opponent to take advantage of than the more naive strategy of  $q=2$ .<sup>11</sup>

To check the NE, first note that although the equilibrium price of energy,  $p$ , is \$20/MWh at node 1, an increase in output of  $>0$  by either player will send this price plummeting to \$0/MWh without affecting the profit of either player. This is because, with TCCs matching output, generators effectively receive the price at Bus 2 independent of the price at Bus 1. Although this ambiguity in price (\$20 vs \$0/MWh) is annoying, it is of no consequence because the distribution of congestion rent is unambiguous. In a complete reversal of our previous conclusion, the TCC owners capture all of the congestion rent and the generators capture none.

Although the generators end up with the TCCs, it is important to see that they do not capture the congestion rents. This would be true even if the TCCs turned out to pay \$20/MWh. In this case \$15/MWh would just go to cover the cost of generation and the other \$5/MWh would go to cover the cost of the TCCs. Thus generators just break even while the initial owners of the TCCs make \$5/MWh off the sale of their TCCs to the generators. It does not matter to them whether they are paid congestion rents directly by the ISO or by the generator who purchased these rights.

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<sup>11</sup> This is because either generator can use the strategy  $(P=5.1, t=4, q=2.1)$ . With this strategy it will secure all 4MW of TCCs and will make a profit of  $4 \times (20 - 5.1) - 2.1 \times 15 = 28.1$ . This demonstrates the problem with fixing  $q$  before the outcome of the TCC auction is known and explains why we use a two stage game.

We now check the Nash equilibrium by evaluating changes in the payoff function caused by increases and decreases in each strategic variable. Note that because initial TCC owners capture all of the congestion rent in the equilibrium, each generator makes a profit of zero. Now consider output,  $q$ . A tiny increase in  $q$  causes the first payoff term to change from  $(20-15)q$  to  $(0-15)q$ . This change is offset by an equal increase in the third term (TCC revenue) but as  $q$  grows the first term becomes increasingly negative while the second two terms stay constant. Thus, increasing  $q$  decreases profit. Decreasing  $q$  simply reduces the \$5/MWh profit that is being made on  $q$  without any offsetting affect.

Increasing  $P$ , say to 5.1, will increase the TCC purchase,  $T$ , to 4, and this in turn will increase  $q$  to 4. The result is a profit of  $(20-15) \times 4 - 5.1 \times 4 + (20-20) \times 4$  which evaluates to  $-0.4$ . (Recall that the payoff function is  $(p-15)q - P \cdot T + (20-p)T$ .) Reducing  $P$  results in no TCCs, no output and zero profit, which is not an improvement. Lastly consider the TCC quantity bid,  $t$ . It cannot be increased because of auction rules, and decreasing it simply results in fewer TCCs, less output, and still a profit of zero. This concludes a fairly thorough check of the Nash conditions. A complete check produces the same conclusion.

### 3.3 *Is There a Nash Equilibrium with Zero-Price TCCs?*

Because a game can have more than one NE, we must now check whether a zero-price TCC equilibrium could exist in TCC game. Although the energy-only model had an infinite set of Nash equilibria, we will select the symmetric one in which both firms produce  $q=2$  for our test of the current game. Is there an equilibrium of the TCC game with these characteristics? If such an equilibrium existed it would specify both players' strategies as  $(P=0, t=4, q=2)$ . In this case both generators make a profit of  $(20-15) \times 2 - 0 \times 0 + (20-20) \times 0 = 10$ . But either could bid  $(P=1, t=4)$  in the TCC auction and buy all of the TCCs for \$4/h. Then it could set  $q=2.1$  and make a profit of  $(0-15) \times 2.1 - 1 \times 4 + (20-0) \times 4 = 44.5$ . So the proposed equilibrium is obviously not an equilibrium.

Although omitting the TCCs market is sufficient explanation for the conclusion that TCCs have no value, there may be a deeper reason for this conclusion. This reason can be found in Oren's first equation which specifies "the optimization problem defining the best response of supplier  $i$ ." Simplifying this condition to the two-generator case we have, for  $i = j$ :

$$\begin{aligned} \text{Max}_{q_i} \quad & B(q_i | q_j) \\ \text{subject to:} \quad & 0 \leq q_i \leq K \text{ \& } q_j \end{aligned} \tag{6}$$

The first line is just the standard Nash condition, but the second condition is left unexplained. It is a side constraint on the players' moves and has the interesting property of preventing the

generators from competing the price down even if a player were to choose to defect from a Nash equilibrium. Thus in Oren's formulation, it is impossible for the generators to bid the price down below \$20/MWh even if they wish to, and thus impossible for them *not* to capture the full congestion rent. This formulation makes Oren's result needlessly tautological, because, as we have seen, the energy-only game produces exactly the set of NE that he found even without this condition.

If this condition is imposed on the TCC game as Oren suggests it should be, then of course generators will again capture all of the congestion rent.<sup>12</sup> It is impossible for them to do otherwise. It should be noted that the problematic second condition, while not explained in his paper, has been identified by Oren as a condition that is allowed only in "GNE" games which are so non-standard that they are not mentioned in any current game-theory text. Such a condition is not allowed within a strategic game as defined by Fudenberg and Tirole (1990, p. 4). This condition is discussed at more length in the Appendix, with the conclusion that even within the GNE game framework this condition is inappropriate for the TCC game.

### 3.4 *Checking for Other Nash Equilibria*

In the energy-only game we found a continuum of NEs, so it is natural to suspect the same type of outcome in our TCC game. In particular we might expect that a pair of strategies such as S1: ( $P=5$ ,  $t=3$ ,  $q=T$ ) and S2: ( $P=5$ ,  $t=1$ ,  $q=T$ ) would be another NE. To see that this is not the case, first note that this pair of strategies would provide each player with a payoff of zero. But player 2 could do better with the following strategy: ( $P=0$ ,  $t=4$ ,  $q=T$ ). At an offer price of zero, player 2 will only get "left over" TCCs which in this case will amount to 1 MW. At a price of zero, this is a bargain. Player 2 can now earn \$5/MWh on 1 MW of production. Although this does not constitute a proof that no other equilibria exist, such a conclusion appears extremely likely.

### 3.5 *Summary*

We have now shown that, at least in one simple TCC game, the congestion rent is captured entirely by the TCC owners as it should be and not by the generators. The value and price

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<sup>12</sup> It is not obvious from Oren (1997a) how he would apply this side constraint in the presence of tradeable TCCs. But, when presented with the possibility that a generator would buy TCCs and defect from his proposed (2,2) equilibrium by increasing output to 3, he responded that such a move is not allowed under the Cournot-Nash assumption and that the only moves permitted would be to reduce output unilaterally. (Email communication April 19, 1997).

of TCCs is determined not by the revenue from a nodal price difference but by their usefulness in bargaining over which Nash equilibrium will be realized. It is also important not to place too much emphasis on our finding of a high price at the congested node. If either generator increased its output by , the price would fall to zero, but nothing material would change. All parties would be exactly as well off as before, and the same power would be generated and flow on the lines. This indicates that our model has insufficient realism to make a sensible price prediction, but it does give firm results on the important variables.

## 4 Why Generators May Retain Market Power

The TCC game completely reversed our results on the energy-only game. Congestion rents were completely retained by TCC owners instead of going 100% to the generators. But this result is too good to be general. Faced with the complete inelasticity of a congested line, it must be possible for generators to capture some congestion rents under some circumstances. As noted earlier the main circumstance of importance is the relative capacity of generators compared with the transmission line. A second fact is the relative quantity of TCCs that are available for purchase by generators. We will consider first one and then the other.

### 4.1 Limited Generation Capacity

One would expect that as total generation capacity,  $Q$ , decreases towards line capacity,  $K$ , market power would increase. Certainly we know that when  $Q < K$  the price at Bus 1 rises to the price at Bus 2 because the line cannot be congested and the two busses will form a unified market. When  $Q$  is just slightly bigger than  $K$ , then any firm can decongest the line at very little cost by reducing its output slightly. In this circumstance, cooperation should not be hard to achieve. On the other hand, if no firm can bring total output below  $K$  on its own, we would expect much less cooperation, and that is in fact what was found in the above example.

Much can be learned by working a second example, similar to the first in all respects except for the capacity of the two generators which we will reduce from 4MW to 3MW. Let us begin this analysis by checking first the previous  $P=\$5$  equilibrium and then the  $P=\$0$  non-equilibrium.

#### *Why TCC Owners Cannot Capture the Full Congestion Rent*

Assume again that both generators play ( $P=5$ ,  $t=4$ ,  $q=T$ ). In this case, as before, we will find  $T=2$  and thus  $q=2$  for each generator, and as before profit will be zero. But either generator can, on its own, do better than that. Either generator can play ( $P=0$ ,  $t=0$ ,  $q=1$ ), and because the other generator can play no more than  $q=3$ , the line is guaranteed to be free of congestion and the price at Bus 1 will be  $\$20/\text{MWh}$ . In this case the generator in question will make a profit of  $(20-15) \times 1 - 0 \times 0 + (20-20) \times 0 = 5$ . Since this payoff is always available, the NE must provide each generator with at least this payoff. Thus the generators will each capture at least  $\$5/\text{h}$  of the  $\$20/\text{h}$  of total congestion rent. The previous NE does not provide this payoff because the price of TCCs,  $P=5$ , is too high.



### *Why Generators Cannot Capture the Full Congestion Rent*

Assume now that TCCs are worthless and therefore can be bought for  $P=0$ . Assume also that we are at a NE of  $q=2$  for each generator and so each earns a profit of \$5/MWh for a total of \$10/h. Now either generator can buy 3MW of TCCs for zero cost and ensure itself a profit of  $3 \times 5 = 15$ , which is 5 more than without the TCCs. (This profit will be earned whether or not the line is congested.) Since in any proposed equilibrium one or the other generator will have an output of  $q=2$  or less, whenever  $P=0$  one generator can always do better by at least \$5/h. Thus TCCs must have value.

### *The Nash Equilibrium with Reduced Generation Capacity*

We have now shown that generators can capture some of the congestion rent (at least \$5/h each) but that TCCs cannot have zero value. So TCC owners will also capture some of the congestion rent. Because either player can make a profit of 5 on its own, one might guess the NE to be  $(P=2.5, t=4, q=T)$ . This strategy is similar to the one we found before and again implies  $q=2$  for each generator and a Bus-1 energy price of  $p=20$ , so profit is 5. This means that neither player can do better on its own by ignoring TCCs and just producing  $q=1$ . But we must look closer. What if one generator played  $(P=2.6, t=3, q=T)$ ? It would win 3 MW of TCCs at a price of \$2.6/MW in the TCC auction and then produce  $q=3$  for a profit of  $(20-15) \times 3 - 2.6 \times 3 + (20-20) \times 3 = 7.2$ . Clearly this is a better strategy, and we have not yet found the Nash equilibrium.

On the other hand, if both players play  $(P=2.6, t=4, q=T)$ , then profit will fall below 5 and either player can do better independently with  $(P=0, t=0, q=1)$ , which earns a profit of 5. This situation strongly indicates that there is no pure-strategy NE. But Nash has assured us that there is always an NE so we must look for a mixed-strategy NE.<sup>13</sup> **Figure 3** shows the result of computing this equilibrium numerically.<sup>14</sup> Competition has bid the price of TCCs up above zero and forced the average profit down almost to what each player can make on its own. The random strategy used by each player is  $(\tilde{P}, t=3, q=T)$ , where  $\tilde{P}$  is the bid price for TCCs selected according to the probability distribution shown in **Figure 3**. The average value of  $\tilde{P}$  is \$2.24/MWh but this is less than the average paid for a MWh of TCCs because the winning

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<sup>13</sup> As before a NE is guaranteed provided bidding is restricted to an integer number of kW for  $q$  and  $t$ , and to an integer number of cents for  $P$ . Without this restriction, the discontinuity in the payoff function make existence problematic.

<sup>14</sup> The sawtooth effect is almost certainly an artifact of the way in which a continuum of strategies was approximated. The interval from  $P=0$  to 5 was represented by 101 evenly spaced values. The payoff function that defines this game is  $B(P_1 | P_2) = 3 \times (5 - P_1)$  if  $P_1 > P_2$ ,  $B(P_1 | P_2) = 2 \times (5 - P_1)$  if  $P_1 = P_2$ , and  $B(P_1 | P_2) = 5 - P_1$  if  $P_1 < P_2$ .

bidder will buy 3 MW of TCCs while the loser will purchase only 1 MW. The average price paid for a MW of TCCs is \$2.486 and the expected profit of each generator is \$5.028. This is very little different from the profit that each can guarantee on its own.

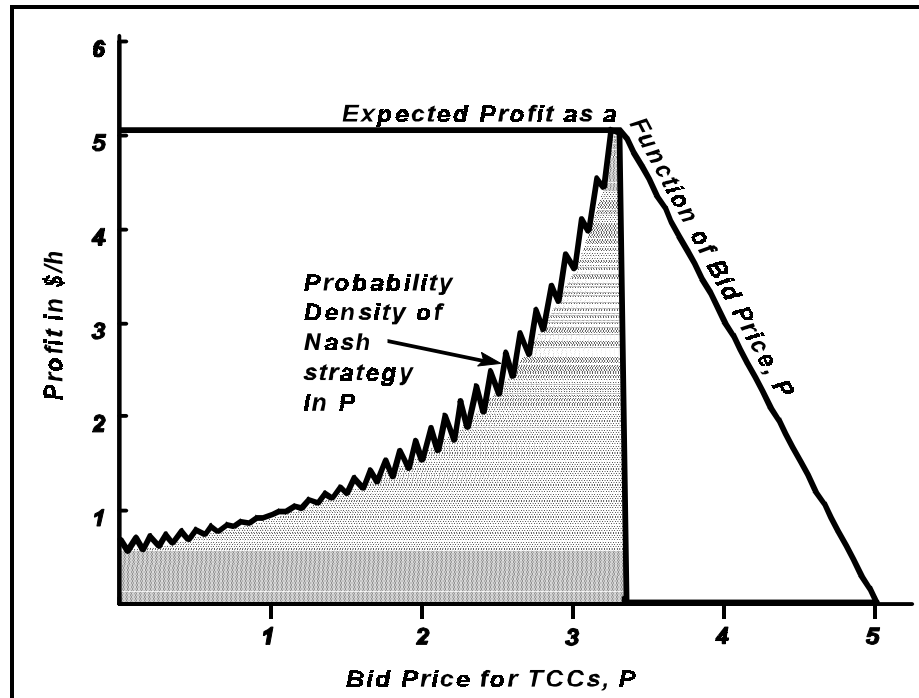


Figure 3. Mixed Strategy and Profit When Output Capacity = 3.

The important conclusion to draw from this example is not that a mixed strategy is used but that TCCs do retain considerable value. Nonetheless, generators did capture some of the congestion rent. Does this partially confirm the result of the energy-only model? Yes, but hardly in a surprising way. What we have found is that when only two generators compete in a market where the demand function is completely inelastic they will exercise some market power.

#### *A More General Result*

The argument given above showing that generators could capture at least some congestion rent suggests a more general result. That argument relied on the fact that a generator could forego the purchase of TCCs and, by limiting its output, ensure that the line was not congested. It would thereby guarantee itself (and all others) the price at the remote bus. It would thereby capture some congestion rent and earn a positive profit. This argument fails

under an easily defined circumstance. In this case the generators will capture none of the congestion rent and TCC owners will capture it all. Our result is as follows.<sup>15</sup>

**Result:** If TCCs are available to the extent of the line capacity, and if  $TC$  is total generation capacity,  $C_{MAX}$  is the capacity of the largest generator, and  $K$  is the outgoing line capacity, then TCCs will completely eliminate the market power of generators if and only if  $C_{MAX} < TC - K$ .

## 4.2 *Insufficient TCCs*

Another way in which generators may gain market power is through the insufficient availability of TCCs. That is, there could be too few TCCs to cover the capacity of the line. Assuming that TCCs are allocated by the ISO according to the method proposed by Hogan (1994) and in more detail by Bushnell and Stoft (1996), it is a little hard to see how this would come about. According to these proposals, the ISO will allocate TCCs to the extent of feasible flows on the network. A shortage cannot be caused by some TCCs being oriented in the wrong direction for this only increases the number of TCCs available in the right direction. (The allocation rule for TCCs simply restricts their vector sum to correspond to a feasible distribution.) It could be that some non-generator TCC owner would decide not to sell his TCCs, but this runs the risk of finding they have zero value because the nodal price difference is zero. Nonetheless let us analyze the problems caused by a shortage of TCCs.

### *A Model with Missing TCCs*

To simplify we return to our first TCC model in which each generator had a capacity of 4MW, but we now reduce the available TCCs from 4 to 3MW. In this game the most obvious NE is ( $P=5$ ,  $t=3$ ,  $q=T+0.5$ ) for both generators. In this case each generator will gain 1.5MW of TCCs and generate  $q=2$ MW of output for a profit of  $(20-15) \times 2 - 5 \times 1.5 + (20-20) \times 0 = 2.5$ . The argument that this is a NE exactly parallels the argument for the NE in the first TCC game. The interpretation of this situation is clear. TCCs are sold and valued at their correct price, but because only 3MW of TCCs participate in the auction, only 3/4 of the congestion rent is captured. In this game only the TCCs that participate in the auction are correctly rewarded. In a more realistic model, the price at Bus 1 might well be depressed by competition, and so non-participating TCC owners might also profit.

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<sup>15</sup> I have been told, but cannot confirm, that Alfred Marshall published a very similar result in *Industry and Trade* (1919).

In order to understand the importance of the production strategy  $q=T+0.5$ , it is useful to consider why  $(P=5, t=3, q=2)$  is not a Nash equilibrium. As noted earlier this allows either generator to utilize the strategy  $(P=5.1, t=3, q=2.1)$  and earn a profit of  $(0-15) \times 2.1 - 5.1 \times 3 + (20-5.1) \times 3$  which is 13.2 and is consequently much better than using the proposed Nash strategy. The importance of this observation is that it demonstrates that the portion of output not covered by TCCs cannot be divided in some arbitrary but constant way. It must be divided according to some rule that is based on the outcome of the TCC market. There are many such rules.

While the  $q=T+.05$  rule may be the most obvious, other similar rules can also be used to form a NE. For example the following is a NE.

Strategy 1:     $(P=5, t=3, q=T+0.1)$   
 Strategy 2:     $(P=5, t=3, q=T+0.9)$

This is exactly the ambiguity we found in the model before TCCs were introduced. Now that ambiguity reappears but is confined to the output region that is not covered by TCCs.

A deeper understanding of why the price of TCCs is 5 in this NE comes from considering the possibility that one generator might sell a fraction of a TCC to the other. If the rule for dividing up the *uncovered portion* of line capacity is that player 1 gets 0.1MW and player 2 gets 0.9 MW, then shifting 0.1 MW of TCC from generator 1 to generator 2 will also shift 0.1 MW of output from 1 to 2. Because getting to sell additional output is worth exactly \$5/MWh so is the TCC that controls that shift in output.

Another reasonable NE is for each player to choose  $(P=5, t=3, q=4T/3)$ . This rule splits the uncovered line capacity in proportion to the amount of TCCs owned. In this case each unit of TCC purchased increases output by  $4/3$  so TCCs are worth  $(4/3) \times \$5/\text{MWh}$ . Since there are 3MWs available, they have a total value of \$20/MWh. So, in this case, the TCC owners capture the full value of the congestion rent on the line. This covers the most plausible Nash equilibria, but we are still left with considerable uncertainty as to exactly how this game will play out. Nonetheless we have every reason to expect that TCC owners will capture a reasonable share of the congestion rent.



## 5 Congestion in a Three-Node Network

We have shown that TCCs can mitigate market power in a one-line network, but does this result generalize to more realistic networks which typically allow loop flow? While Oren considered a market for “active transmission rights” in a three line setting he did not consider a market for TCCs and so makes no argument to the contrary. However he does assert that without any market for transmission rights, a three node network with one congested line will result an inefficient dispatch in which generators capture all of the congestion rent. This last claim goes beyond what was claimed in the one-line case and so deserves attention. We will consider a specific example of this claim.

Consider the three-bus network with  $2¢/\text{kWh}$  generation at Bus 1,  $4¢/\text{kWh}$  generation at Bus 2, and demand of  $21 - 4P_3$  at Bus 3, where  $P_3$  is the local price. Line 1-2 has a capacity limit of 1, and the other two lines have no capacity limit. The lines all have the same impedance. For this example Oren claims the following Nash equilibrium.

$$\begin{aligned} P_1 &\ni P_2 \ni P_3 \ni 4¢/\text{kWh} \\ Q_1 &\ni 4, \quad Q_2 \ni 1, \quad Q_3 \ni 5 \end{aligned} \quad (7)$$

Notice that this dispatch is feasible because  $1/3$  of the power flow from Bus 1 to Bus 3 flows through line 1-2, but the excess flow is exactly canceled by a  $1/3$  of the flow from Bus 2 to Bus 3. Notice also that by our previous convention, line 1-2 is not congested. So the three nodes should have the same price which they do.

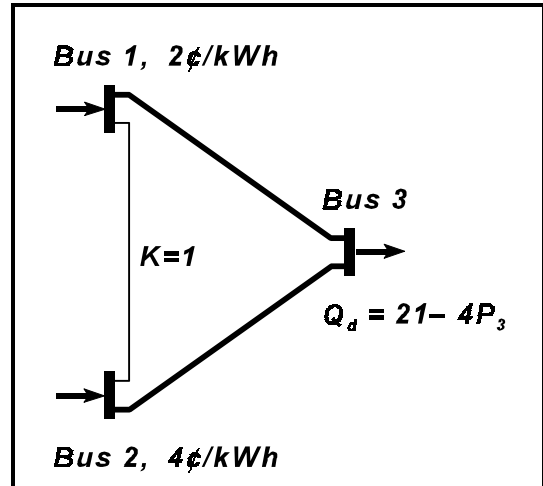


Figure 4. A Three-Line Example with One Congested Line.

This “equilibrium” is very favorable to generators at Bus 1 because they are paid well above marginal cost and thus earn a profit of  $2¢/\text{kWh}$ , while generators at Bus 2 earn nothing. To check this “equilibrium” we must define the Cournot game, which we do as follows. Generators each bid a quantity. If the bids are feasible then the price at all nodes is set by the demand function, and all generators produce at their bid levels as is illustrated by the above “equilibrium.” If the bids at Bus 2 are insufficient to render the bids at Bus 1 feasible the line 1-2 is congested. In this case the marginal value of additional bids at Bus 1 is zero and at Bus 2 is twice  $P_3$  because each additional kW of generation at Bus 2 allows two additional kW to be consumed at Bus 3. This conforms to Oren’s statement that “when both supply nodes generate, the selling price at Node 3 (under optimal dispatch) is the average of the bid prices at DEARGEN and CHEAPGEN” (p. 72).

## 5.1 *Not a Nash Equilibrium*

With the game specified we can now ask, could the generators at Bus 2 do better? We find that the answer is clearly yes. If any generator at Bus 2 decreases its output by 1, the price at Bus 2 will double to 8¢/kWh. What we see is a struggle between generators at Bus 1 who want the line uncongested so that they will be paid the price at Bus 3, and the generators at Bus 2 who want the line congested so that they will be paid twice the price at Bus 3 because their output is needed to utilize the power bid by Bus-1 generators. Clearly we do not have a Nash equilibrium, so it is premature to conclude anything about who collects the congestion rent or the inefficiency of the dispatch.

Unfortunately, as demonstrated in the appendix, there is no pure-strategy Nash equilibrium for this game. Because of the difficulty in computing this mixed equilibrium nothing is yet known about it. However some insight into this market can be gained by introducing the possibility of arbitrage. This is a realistic assumption in most markets and is particularly appropriate in the electricity market given the rate at which power marketers have been registering with FERC. So let us present an arbitrager with Oren's equilibrium and see if it can make a deal.

## 5.2 *Arbitrage Without Transmission Rights*

The keys to arbitrage are the excess capacity at Bus 1 along with a hefty markup. The efficient dispatch of the market requires that  $Q_1 = 6$  so, if efficiency is possible, there must be at least 1 firm with 33% excess capacity.<sup>16</sup> This firm would be willing to sell full-capacity output at any price above 3.33¢/kWh, so let us say the arbitrager offers it 3.5¢/kWh. Generators at Bus 2 are making no profit so they would all be willing to sell any amount of their output at a price of 4.1¢/kWh. Thus the arbitrager can buy power at an average price 3.8¢/kWh and sell it at Bus 3 for 3.9¢/kWh. Clearly this undermines the proposed equilibrium. But what is new is that it moves us in a more competitive direction. It increases output and reduces the price at Bus 3.

One detail of the above transaction must be filled in. In order to make such trades, the arbitrager will rely on contracts for differences (CFDs) relative to the price at the relevant busses. Thus if the price at Bus 1 turns out to be 0, and the price at Bus 2 turns out to be 8¢, the arbitrager will have to pay the generator at Bus 1 the full 3.5¢ but will be paid 3.9¢ ( $8¢ - 4.1¢$ ) by the generator at Bus 2, thereby assuring both generators of their contract price. In this case the price at Bus 3 would be 4¢ so the arbitrager would pay the load 0.1¢ to bring its price down to 3.9¢ as promised. The net result for the arbitrager is a profit of 0.1¢/kWh

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<sup>16</sup> The efficient dispatch is  $Q_1 = 6$ ,  $Q_2 = 3$ ,  $P_3 = 3¢/\text{kWh}$ ,  $Q_3 = 9$ .

as planned. This strategy of hedging requires the arbitrager to buy equal quantities from Bus 1 and Bus 2 which explains our assumption that the arbitrager did so.

Although we have not found the equilibrium of this expanded model, we have a strong indication that it is much more competitive than Oren's proposed equilibrium. Oren may well have been aware of this arbitrage possibility as is indicated by his *Energy Journal* article which contains a curious parenthetical caveat, "(no side payments are allowed)," in his description of the three-node Cournot model. Note that the competitive pressures of this arbitrage spring not from transmission right of either variety but from the simple contract for differences. Note also that this arbitrage could be carried out by any of the players, generators or loads, in the present model.





## **6 Conclusions**

We have extended previous work on the value of financial transmission rights (such as TCCs) in the presence of generator market power. In the process, we have found that such rights can be extremely effective in limiting the market power of generators. It had been demonstrated that for a one-line network, in the absence of purchasable TCCs, the infinite set of Cournot-Nash equilibria would all allow generators to capture the congestion rent completely. Because there would be great rivalry between generators over the choice of equilibria, this finding leaves ambiguous the actual outcome of the Cournot game, but it suggests significant generator market power. In the extreme this could make untradeable TCCs worthless.

By extending this analysis to include the purchase of TCCs by generators we have found that TCCs gain a well-defined value that allows their owners to capture much or all of the congestion rent on the exporting line. In the process, the market power of the generators is reduced or eliminated. The extent of this limitation depends on the extent to which generation capacity exceeds export capacity and on the size of individual generators. It is suggested that market power will be reduced to zero whenever total generation capacity exceeds market capacity by more than the capacity of the largest generator, provided TCCs are available to cover the full capacity of the line.

Lastly we examined the extension of these arguments to networks with more than one line. In this case we found previous claims of a pure-strategy Nash equilibrium to be in error, and evidence that arbitrage would prevent the generators at the congested bus from capture all congestion rents. Significantly this arbitrage depended only on contracts for differences and not on transmission right of any type.



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## Appendix

### *GNE Games and the Feasibility Constraint*

In a real network, generators are not allowed to generate beyond the network's capacity to transmit. For instance in the one-line network with a capacity limit of 4 MW the two supply generators would be required to keep their total output below this limit. There is a standard and a non-standard approach to modeling this situation.

The standard game theory approach would allow the two generators to make independent decisions on output or bidding and penalize each if they jointly violated the constraint. One model would pay zero for all output whenever the total exceeded 4 MW, and the ISO would dispose of the extra power. Another model would let each generator bid and then let the ISO dispatch according to feasibility and bid prices and quantities. We can produce a Cournot model by restricting the generators in either model to pure quantity moves. We have adopted the first model because of its simplicity and its similarity to standard Cournot models. Because this specification allows independent action by the two players and specifies consequences for all possible combinations of moves, it translates directly into a game as it has been defined from Von Neumann and Morgenstern (1944) to Fudenberg and Tirole (1990).

A non-standard game theory approach to this problem has been adopted by both Oren (1997a and 1997b) and Hogan (1997). Although not acknowledged in his paper, Oren's first equation contains a condition that is not allowed under standard game theory. The game definition that both rely on is known variously as a "generalized Nash equilibria (GNE) game," a "social equilibria game," or a "pseudo-Nash equilibria game."<sup>17</sup> This definition of a game allows one player's set of legal moves to depend on the other player's choice of move, even though they both move simultaneously. This allows the definition of a game in which the players' moves automatically satisfy the feasibility constraints. Unlike a standard game, this specification does not model some well defined procedure but instead is meant to mimic an unexplained negotiation that takes place between the two players.

Some who work with GNE games question the use of such games to model real situations and claim they are "...only useful as a mathematical tool to establish existence theorems ..." (Ichiishi, 1983). But both Oren and Hogan cite Harker (1991) who claims that GNE games can model real situations but notes that for this to work "players must be given incentives to obey these constraints on their activities." In other words the constraints must correspond not simply to a notion of what outcome we expect but to a penalty function that will force a negotiated outcome that conforms to the constraint. In this case the GNE game is a shortcut

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<sup>17</sup> Hogan references Harker (1991) and Oren has cited this paper in a personal communication to the author.

around modeling the penalty function in a standard game. With this in mind, let us consider the current use of GNE games.

Oren uses a GNE constraint on outputs that constrains them to satisfy  $q_1 + q_2 \leq 4$  in our energy-only game while specifying that so long as they do, the price at their node will be the uncongested price. He then concludes that TCCs will be “preempted by the strategic bidding of the generators.” In this case the constraint does not interfere with the determination of Nash equilibria, but it renders meaningless the claim of “strategic bidding” by allowing the generators no possibility of bids that fail to capture the congestion rents. If the GNE constraint is replaced with a standard game model, the same Nash equilibria can be found, and the generators can reasonably be described as acting strategically. In this case the GNE shortcut is successful at identifying the Nash equilibria because players have a sufficiently strong incentive to conform, but it distorts our view of the situation by making it appear that generators could not compete the price down if they so desired.

Once TCCs are introduced, this GNE constraint violates Harker’s dictum on incentives. In the energy-only game if player 1 chooses  $q_1 = 2$ , then player 2 will know that choosing  $q_2 > 2$  will cause price to drop to zero. Thus player 2 will have an incentive to obey the constraint. In the TCC game, player 2 may own a TCC for 3, in which case it will have an incentive to violate the constraint thereby forcing the local price down and profiting from its TCC. This use of the GNE constraint contradicts the rational behavior that we had hoped to model. It must be admitted that it is possible to rescue this GNE constraint by specifying that supply price equals zero whenever total supply equals 4. This allows player 2 to force the price down to zero by setting  $q_2 = 2$ . From this we must conclude that a careful approach to GNE modeling can yield games that give correct results.

Nonetheless, because a GNE game models an unspecified process of negotiation, the only way we can be sure that such a game serves as an accurate model is by carefully comparing it to the incentives of the real situation. But these incentives are simply payoff functions applied to players’ strategies that are chosen independently (without a GNE constraint). Specifying these payoffs and strategies results in the definition of a standard game. Because this standard game corresponds much more closely to the real situation, its faithfulness is more easily verified. Since a correct GNE formulation will have the same Nash equilibria as the standard game, the standard formulation should always be our starting point. If a GNE constraint is needed to aid in the solution of the game (a tactic Harker claims is useful), then the GNE constraint should simply be applied to the definition of the Nash equilibrium, not to the definition of the game. A close reading of Hogan’s specification shows that this is in fact the approach he has taken, and that he has not actually specified a GNE game.

By using a standard game to model Cournot competition, we lose nothing and gain a faithfulness to reality that makes checking the specification far easier. If a GNE condition is added to the definition of a Nash equilibrium in order to aid computation, it is easy to check

any discovered equilibria against the more faithful standard game, and it allows the possibility of discovering NE that satisfy the standard game but not the GNE game.

### *A Faulty TCC Market*

A faulty TCC market has been proposed as a third way in which generators could capture the congestion rents. This is not a real possibility unless TCC owners are forced to participate in the badly designed market, an unlikely possibility. If such a mechanism were implemented without coercion, either by regulators or by the private market, TCC owners would simply bypass it and sell their TCCs in an unstructured over-the-counter market. Still, it is instructive to consider how a faulty market could reduce the value of TCCs.

The second-price auction has been suggested as a way in which TCCs could become valueless. To demonstrate this we return to our first TCC game but with a change in rules for the TCC auction. Now instead of charging each bidder its bid price, we charge each bidder only the price of the first losing bid. (This is called the second price because the auction is usually defined for a situation where there is one winner with the highest bid, and the winner pays the price of the second highest bid). If there is no losing bid, then the losing bid price is taken to be zero. With these new rules we find that there are new Nash equilibria that gives TCCs a value of zero. In one such equilibrium both generators choose  $(P=25, t=2, q=2)$ , and both bids for TCCs are successful. In this case there is no losing bid so the price of TCCs falls to zero.

The value of \$25 has no special importance. Its role is simply to be large enough to deter the other player from trying to buy any more TCCs. This strategy is effective. Notice that either generator would profit greatly if it could buy all 4 MW of TCCs for a price of zero. But if either increases its bid by even the smallest amount the total of the bids will exceed 4 MW and there will be a portion of one bid that loses. This will set the price for both bidders at \$25/MWh. Now if either bids high and gains more than 2 MW of TCCs, the other will lose on a portion of its 2 MW bid. This portion will have a price of 25\$/MW. This is a losing strategy for both. Thus, the proposed equilibrium is Nash. Although TCCs still have value to generators, their owners are not able to capture any of this value. I claim that TCC owners would soon find another way to sell their TCCs to generators.



*Why the Three-Node Nash Equilibrium Uses Mixed Strategies*

In Section 5 we argued that Oren's proposed Nash equilibrium is undermined by generators at Bus 2 who will choose to curtail output by , in order to congest Line 1-2. In fact, it can be shown that this game has no pure-strategy Nash equilibrium. This is because the cheap generators always back off until the line is uncongested, while the expensive generators always back off until the line is congested. This might lead to an equilibrium in which the expensive generators produced nothing at all. But, in this case, the cheap generators can produce only 3 and the price is thus  $(21 - 3)/4 = 4.5\text{¢/kWh}$ . At this price, any expensive generator can produce at a profit.